The concepts that students begin learning by the 8th grade, including an understanding of fractions, continue to be important concepts in future mathematics classes. When students understand fractions, they carry with them an understanding of prime factorization and a beginning understanding of inverses. When students begin higher-level courses that involve the use of variables, the importance of the previous work becomes clearer. Simplification of rational expressions, order of operations, use of parentheses all become critically important, and misconceptions from those earlier years can wreak havoc.

The teacher has the power and obligation to discover problem areas and fill the gaps (Garlikov, 2000). When one is teaching simplification of rational expressions, one must begin by reviewing simple cases of numeric fractions. As students mature they are better able to grasp the concepts, and a relatively quick (yet deliberate and careful) review should suffice to bring the student back to current material. Once the concept of prime factorization is reviewed, the idea of exponents, and how to interpret them can ensue. Take, for example, the problem of simplifying $\frac{50}{27} \cdot \frac{3}{30}$. A sound way to simplify involves prime factorization:

$$\frac{50}{27} \cdot \frac{3}{30} = \frac{2 \cdot 5 \cdot 5}{3 \cdot 3 \cdot 3} \cdot \frac{3}{2 \cdot 3 \cdot 5} = \frac{\cancel{2}^{1} \cdot \cancel{5}^{1} \cdot 5}{3 \cdot 3 \cdot \cancel{2}^{1}} \cdot \frac{\cancel{5}^{1}}{\cancel{2}^{1} \cdot 3 \cdot \cancel{5}^{1}} = \frac{5}{3^{3}} = \frac{5}{27}$$
 Then problems such as

 $\frac{2x^2}{y^3} \cdot \frac{y}{6x}$ become easier to handle, as the process is the same, using prime factorization.

$$\frac{2x^2}{y^3} \cdot \frac{y}{6x} = \frac{2 \cdot x \cdot x}{y \cdot y \cdot y} \cdot \frac{y}{2 \cdot 3 \cdot x} = \frac{\cancel{2}^1 \cdot \cancel{x}^1 \cdot x}{y \cdot y \cdot \cancel{y}^1} \cdot \frac{\cancel{y}^1}{\cancel{2}^1 \cdot 3 \cdot \cancel{x}^1} = \frac{x}{3y^2}.$$
 It would strengthen the learning

experience to point out that in the above example I used x = 5 and y = 3 to demonstrate the

similarity and complete the lesson in connections. The student is able to relate an easy to follow example to a conceptually more difficult example. With enough practice, the student makes the connection between numerical fractions and rational expressions. It is simply another step to the teaching of more difficult rational expressions involving binomials and trinomials, for example.

How can the teacher fill the gaps without sending the student back for remedial classes? Assuming that the gaps are relatively few or minor, using connections as above is the best approach. Another idea from our discussions is to make tutoring part of the requirement similar to labs in science classes, either in person or better yet, online. This would be an excellent way to encourage special small group or one-on-one help targeting specific misconceptions. In this way the student is supported and gains the knowledge necessary to progress.

Furthermore, studies show that the use of the calculator can help the student learn more concepts by freeing up valuable conceptual learning time (TI, 2007; Barton, 2000; Milou, 1999). At the collegiate level this would be an important additional aid to learning, and could expedite the prime factorization process.

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