

This unit on discovering and filling gaps at the college level focused on teaching the factoring of polynomials. Adults will appreciate a step-by-step approach to polynomial factoring. This will include:

1. Find and pull out the GCF. This is opposite of a simple distributive property, and since the distributive property is multiplication, the undoing of the distributive property is division. To connect to arithmetic, one can break down the terms into their prime factors, circle all that are common to all terms, and whatever is left over goes into a parentheses.

Example (somewhat abbreviated):  $12x^3y - 15x^2y^2 + 27x^3y^2$

$$= 2 \cdot 2 \cdot \underbrace{3}_{\text{common}} \cdot \underbrace{x}_{\text{common}} \cdot \underbrace{x}_{\text{common}} \cdot \underbrace{x}_{\text{common}} \cdot y - \underbrace{3}_{\text{common}} \cdot \underbrace{5}_{\text{not common}} \cdot \underbrace{x}_{\text{common}} \cdot \underbrace{x}_{\text{common}} \cdot y \cdot y + \underbrace{3}_{\text{common}} \cdot \underbrace{3}_{\text{common}} \cdot \underbrace{3}_{\text{common}} \cdot \underbrace{x}_{\text{common}} \cdot \underbrace{x}_{\text{common}} \cdot \underbrace{x}_{\text{common}} \cdot y \cdot y = 3x^2y(4x - 5y + 9xy)$$

2. Binomials: Look for difference of squares. Explain what the vocabulary means and show several examples of varying complexity for the idea of squares. Then show how  $(a+b)(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$  first using single variables and numbers.

Example:  $(x+3)(x-3) = x^2 - 3x + 3x - 9 = x^2 - 9$ . Then, so if we wish to go the other direction, from  $x^2 - 9 = (x+3)(x-3)$ , what can we notice?

Binomials: Look for difference or sums of cubes. The process here would be similar as above, start by multiplying a binomial to an appropriate trinomial to demonstrate the relationship.

3. Trinomials in the form  $ax^2 + bx + c$  with  $a = 1$ . Again, begin with showing two parentheses multiplied together to equal the trinomial. Show several examples, then

discuss what we notice happening between the trinomial and the two parentheses. A complete discussion including reviewing sign multiplication rules should ensue.

Trinomials in the form  $ax^2 + bx + c$  when  $a \neq 1$ . This is a great way to introduce the student to factoring by grouping, used in step 4. Find two numbers that multiply to equal  $ac$ , but add to equal  $b$ . Then separate the trinomial middle term ( $bx$ ) into two terms using the coefficients (the two numbers) found. Now group the first two terms and the last two terms to pull out the GCF of each. Pull out GCF again for the final solution. Example:

$$\begin{aligned} 2x^2 - 3x - 14 &= 2x^2 - 7x + 4x - 14 = (2x^2 - 7x) + (4x - 14) = x(2x - 7) + 2(2x - 7) \\ &= (2x - 7)(x + 2). \text{ The end.} \end{aligned}$$

4. Four terms, factor by grouping. Similar to above, group and factor. Look for parentheses shared and pull it out. Emphasize that this is division, the opposite operation of multiplication, and that we can recuperate the original through multiplication (Nite, 2007).

Learning how to factor polynomials, especially in this technologically advancing age, is important for continuing the development of confident students. Students who are successful at this level of mathematics do gain self-confidence that helps in other areas and future classes.

This is perhaps a critical time for many of the students. Adult students persist in misconceptions, which “must be deconstructed, and teachers must help students reconstruct correct conceptions.” (Allen, 2007). Connecting the factoring of polynomials to arithmetic and particularly to using prime factorization is very helpful. Furthermore, categorizing concepts helps students’ minds stay organized. “Concepts that relate to each other, reinforce each other, and illuminate each other” (Lewis, 2006) aid in conceptual understanding; thus the students do not need to rely on memorization of rules.

**References:**

Allen, G. Donald (2007), *Student Thinking*, <http://www.math.tamu.edu/~snite/MisMath.pdf>

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