Jayne Overgard
Math 646-720
HW 1

1. Pony Problem #1:

Let x = the horizontal distance in miles from (0,0) where the pony diverts toward (6,5).

$D_1 + D_2 = D_T$
$r_1 t_1 + r_2 t_2 = D_T$
$t_1 + t_2 = 1.3$ $\qquad\qquad t_2 = 1.3 - t_1$

$D_1 = x + 6 \qquad D_2 = \sqrt{(6-x)^2 + 5^2} = \sqrt{x^2 - 12x + 61}$

$12t_1 + 10(1.3 - t_1) = 2t_1 + 13 = D_T$

$2t_1 + 13 = x + 6 + \sqrt{x^2 - 12x + 61}$

$12t_1 = D_1 = x + 6$ so $t_1 = \dfrac{x}{12} + \dfrac{1}{2}$

$$2\left(\frac{x}{12} + \frac{1}{2}\right) + 13 = x + 6 + \sqrt{x^2 - 12x + 61}$$

$$\sqrt{x^2 - 12x + 61} = \frac{-5x}{6} + 8$$

$$x^2 - 12x + 61 = \left(\frac{-5x}{6} + 8\right)^2 = 64 - \frac{40x}{3} + \frac{25}{36}x^2$$

$$\frac{11}{36}x^2 + \frac{4}{3}x - 3 = 0$$

$$11x^2 + 48x - 108 = 0$$

$$x = \frac{-48 \overset{\pm}{+} -\sqrt{48^2 - 4\cdot 11\cdot(-108)}}{22} = \qquad \frac{18}{11} \qquad \text{or} \qquad x = -6$$

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At x = -6 the pony would go directly from (-6,0) to (6,5). This path is indicated by the line equation:

y = mx + b   m = 5/12   b =5/2, so

$y := \frac{5}{12}x + \frac{5}{2}$ where $-6 \le x \le 6.$

At $x = \frac{18}{11}$, the path is y=0, where $-6 \le x \le \frac{18}{11}$; then the pony veers left toward (6,5). This second part of the path is indicated by the line equation:

$y := \frac{55}{48}x - \frac{15}{8}$, where $\frac{18}{11} \le x \le 6$

2. Pony Problem #2:

The pony begins at (0,0). We must find all points (x,y) in the field that the pony can end up if the pony first runs down the road, then turns left 90 degrees. The rate on the road is 12 mph, the rate on the field is 10 mph. Total time $t_1 + t_2$ is given as 1 hour.

Let x and y represent the x and y coordinates of all points in the solution set. D_1 = the distance in miles on the road, D_2 = the distance in miles on the field, D_T = total distance;

r_1 = rate on the road = 12
r_2 = rate on the field = 10

t_1 = time on the road
t_2 = time on the field

$D_1 + D_2 = D_T$

$r_1t_1 + r_2t_2 = D_T$

$t_1 + t_2 = 1$

$12t_1 + 10(1 - t_1) = D_T$

$x + y = D_T$
$12t_1 = x = D_1$

$10 - 10t_1 = y = D_2$

$12t_1 + 10 - 10t_1 = x + y$

$2t_1 + 10 = x + y$

$2\cdot\left(\frac{x}{12}\right) + 10 = x + y$ $\qquad$ $\frac{x}{6} + 10 = x + y$

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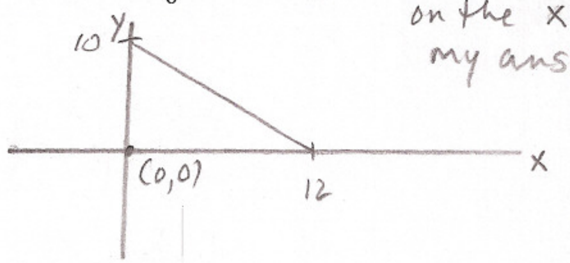
So the solution is $y = \frac{-5}{6}x + 10$ where $0 \le x \le 12$, Because symmetry exists on the x and y axes, I restricted my answer to the first quadrant.



3. Pony Problem #3:

In this problem I began with the same line equation found in Problem #2:

$y = \frac{-5}{6}x + 10.\ 0 \le x \le 12.$ These (x,y) pairs represent the points in the field that the pony can reach if running down the road first at 12mph and then turning left 90 degrees and running at 10 mph for exactly 1 hour total time.

But since the pony can run anywhere in the field, the actual locus of points is the area inside the arcs of circles formed where the center is on the x-axis and the radius is the time in the field. There are an infinite number of these arcs, and so I just needed to find the tangent to the first arc, the one with center (0,0) and r = 10.

Using Geometry and knowing that the tangent line forms a 90 degree angle with the radius at the point of tangency:

$10^2 + a^2 = 12^2$ where a is the length of the tangent segment from (12,0) to the point of tangency.
$a = 2\sqrt{11}$

To find the point of tangency I solved for the system:
$x^2 + y^2 = 100$
$(x - 12)^2 + y^2 = 44$

The first equation is the circle with center at (0,0) and r=10, the second equation is the circle with center at (12,0) and $r = 2\sqrt{11}$

$y^2 = 100 - x^2$ and $y^2 = 44 - (x - 12)^2$

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$100 - x^2 = 44 - (x^2 - 24x + 144)$
$100 - x^2 = 44 - x^2 + 24x - 144$
$100 - x^2 = -100 + 24x$
$200 = 24x$
$x = \frac{25}{3}, \ y = \frac{5\sqrt{11}}{3}$

To find the line equation of the tangent line, we can use that point of tangency and (12,0), another point on the line.

$y = mx + b$

$$m = \frac{5\frac{\sqrt{11}}{3}}{\frac{25}{3} - 12} = \frac{-5\sqrt{11}}{11}$$

To find b: $0 = \frac{-5\sqrt{11}}{11} 12 + b$

$$b = \frac{60\sqrt{11}}{11}$$

$$y = \frac{-5\sqrt{11}}{11}x + \frac{60\sqrt{11}}{11}$$

So the answer is the area in miles^2 bounded by:

the line $y = \frac{-5\sqrt{11}}{11}x + \frac{60\sqrt{11}}{11}$ with domain $\left(\frac{25}{3} \le x \le 12\right)$

the arc formed by $x^2 + y^2 = 100$ with domain $0 \le x \le \frac{25}{3}$ and range $y > 0$

and the x-axis and the y -axis.

Since there is symmetry with respect to both the x and y axes, I restricted my answer to the first quadrant.

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Here is a graph of the answer to #3: