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Assignment 11

The idea of undoing is a very important concept linked to solving equations. The idea in solving equations is to "neutralize", or undo coefficients or numbers somehow attached to the same side of the = sign as the variable so that the variable becomes isolated, and thus solved for.

In solving very simple algebraic equations, one must come to understand that the concept of additive inverse and what it means: $x - x = 0$. Also, the concept of reciprocals: $x * 1/x = 1$. Why are these important? It is because $x + 0 = x$ (the thing attached to x through addition has been neutralized). And $x * 1 = x$ (the thing attached to x through multiplication has been neutralized).

One of the ways this concept of undoing comes up (or should come up) very early on is with teaching addition of signed numbers on a number line. When we begin at zero and move say x units to the right, then move $-x$ units (to the left), we end right back up at zero. We have undone the first thing we did. Subtraction means adding the opposite, or the additive inverse. (<http://www.mathnstuff.com/math/spoken/here/2class/130/c13how2.htm>).

The concept of "undoing" permeates many areas of mathematics. For example, the inverse of $f(x)$ is $f^{-1}(x)$. What does this mean, exactly? It means that

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$. Since the functions are inverses of each other, they "undo" each other.

To undo squares, take the square root. To undo cubes, take the cube root. Exponents are undone by taking the reciprocal of the power to "undo" it. For example, $x^{\frac{-2}{3}} = 64$. To solve,

raise both sides of the = sign to the $\frac{-3}{2}$ power. Hence,

$$(x^{\frac{-2}{3}})^{\frac{-3}{2}} = (64)^{\frac{-3}{2}} = (2^6)^{\frac{-3}{2}} = 2^{-9} = \frac{1}{2^9} = \frac{1}{512}. \text{ So } x = \frac{1}{512}.$$

To undo a square matrix A , we need to find A^{-1} such that $A \cdot A^{-1} = A^{-1} \cdot A = I$. The identity matrix I is such that $A \cdot I = I \cdot A = A$. This process of undoing is used repeatedly in mathematics to work backwards to find solutions.

One uses arcsin and sin as opposite operations. One uses integrals and derivatives as opposite operations. One uses logs to find the value of exponents. Factoring, in a sense, can be thought of as the opposite of the distributive property or expanded form. Often, one must factor to solve.

While the concept of simplifying expressions is an attempt to make an expression into a nicer, neater, smaller-looking package, the concept of solving equations involves “undoing” things to isolate what it is you are looking for. It is the way in which we are able to work backwards, if you will, to solve.